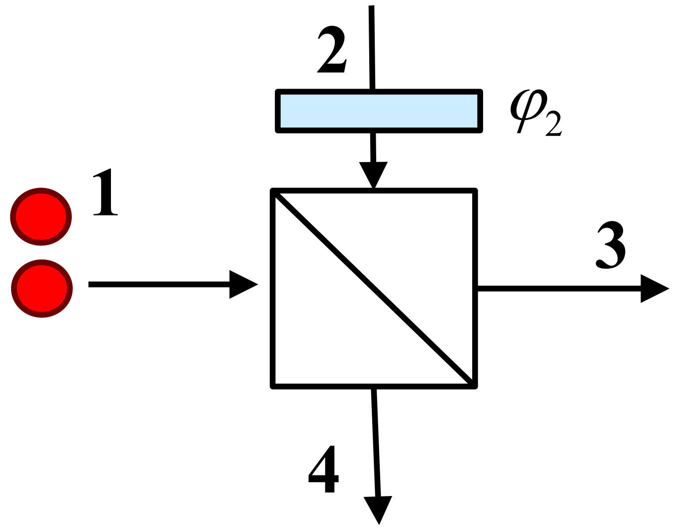
Distribution binomiale: basics

$$(x+y)^2 = x^2 + 2\cdot xy + y^2$$

$$a_1^+ \rightarrow t\cdot a_3^+ + r\cdot a_4^+$$



$$t = \frac{1}{\sqrt{2}} \qquad r = \frac{i}{\sqrt{2}}$$

$$\left(a_3^+ + ia_4^+\right)^2 = \left(a_3^+\right)^2 + 2i\cdot a_3^+ a_4^+ - \left(a_4^+\right)^2$$

Distribution binomiale

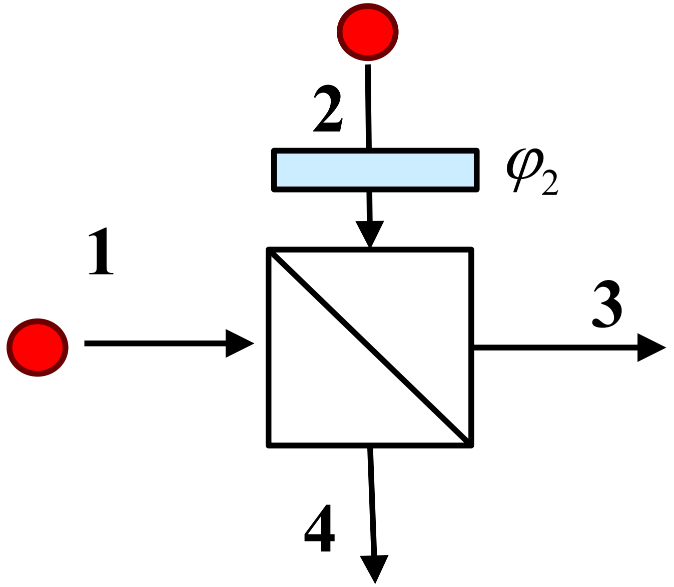
Effet HOM: basics

$$\left(x+y\right)\cdot\left(x-y\right)=x^2+0\cdot xy-y^2$$

$$a_2^+ \rightarrow -\overline{r}\cdot a_3^+ + t\cdot a_4^+$$

$$a_1^+ \rightarrow t\cdot a_3^+ + r\cdot a_4^+$$

$$t=\frac{1}{\sqrt{2}} \qquad r=\frac{i}{\sqrt{2}}$$

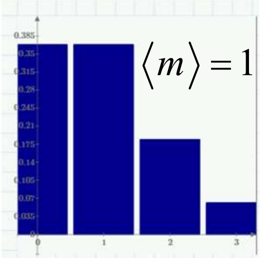


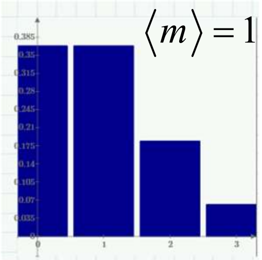
$$\left(a_3^+ + i a_4^+\right)\cdot\left(i a_3^+ + a_4^+\right)=i\cdot\left(a_3^+\right)^2+0\cdot a_3^+ a_4^+ + i\cdot\left(a_4^+\right)^2$$

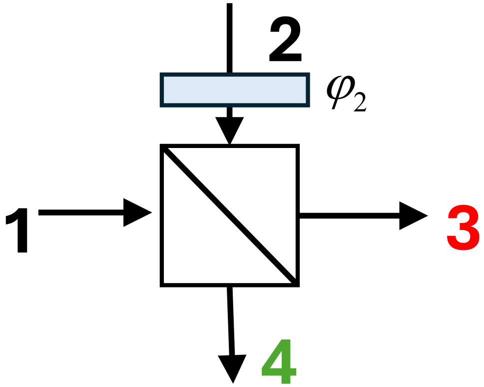
HOM

Exercice 10-1:
Beam splitter et modes cohérents: un mode cohérent à chaque entrée

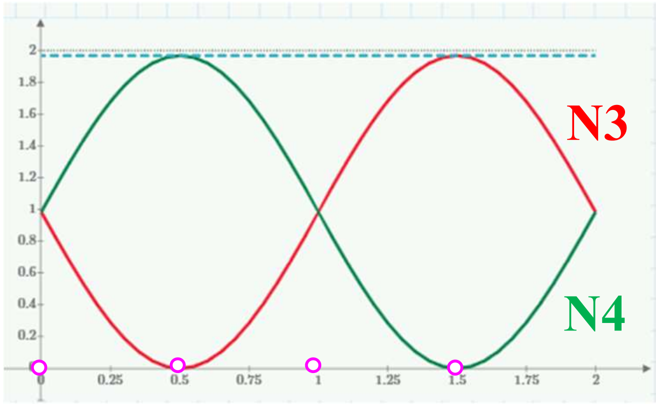
Les modes cohérents
se comportent comme
des rayons laser classiques
??? Pourquoi ???

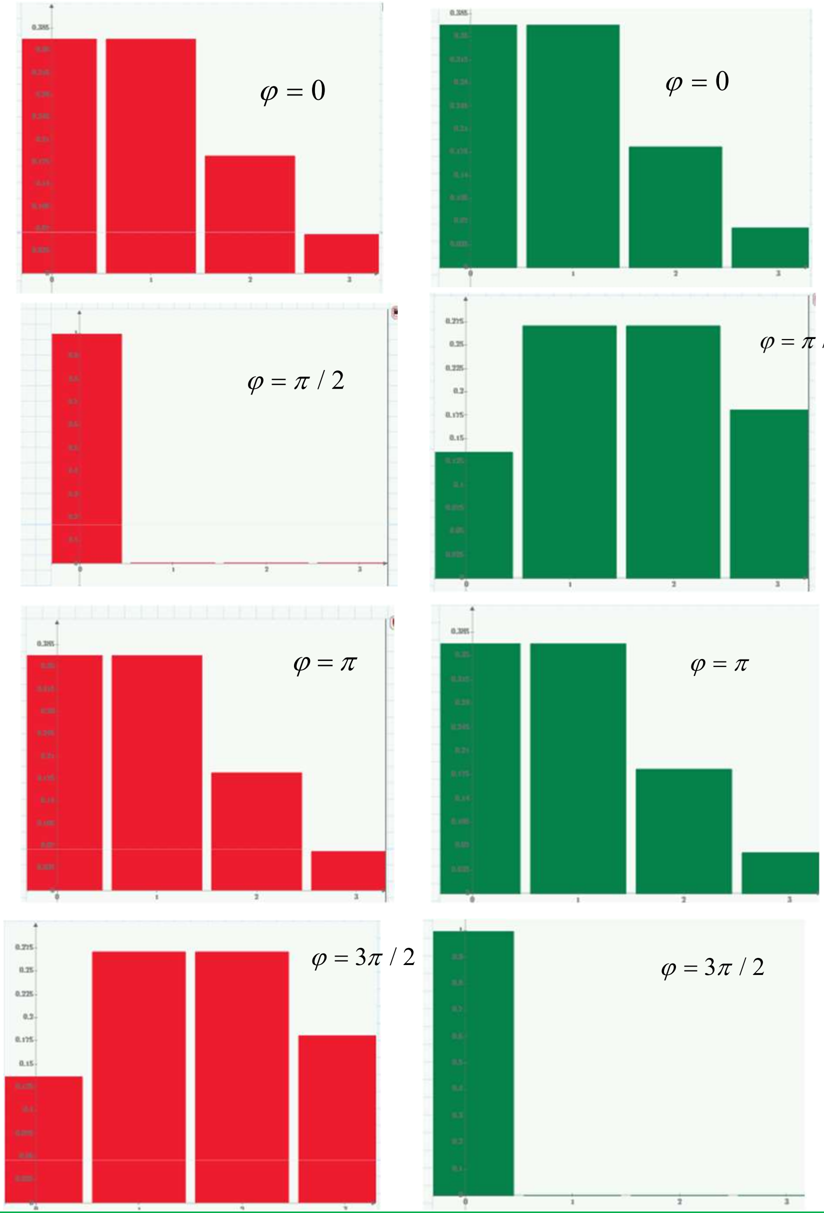




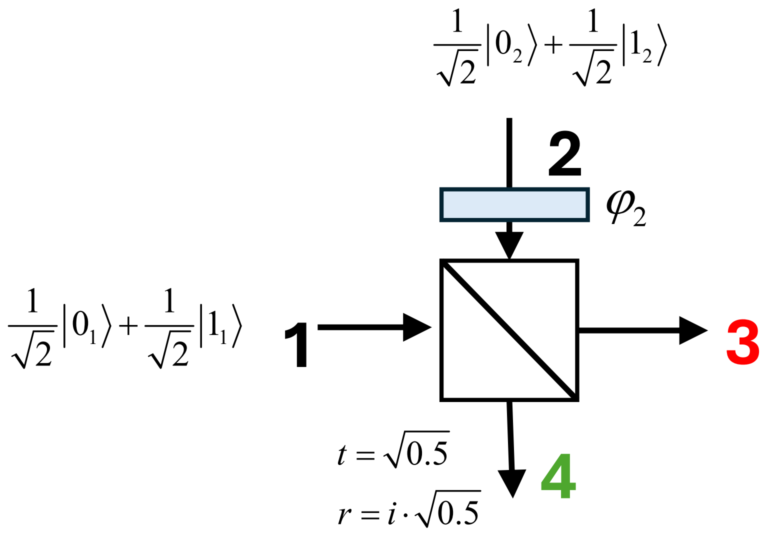


Interférences





Exercice 10-1:
Modèle

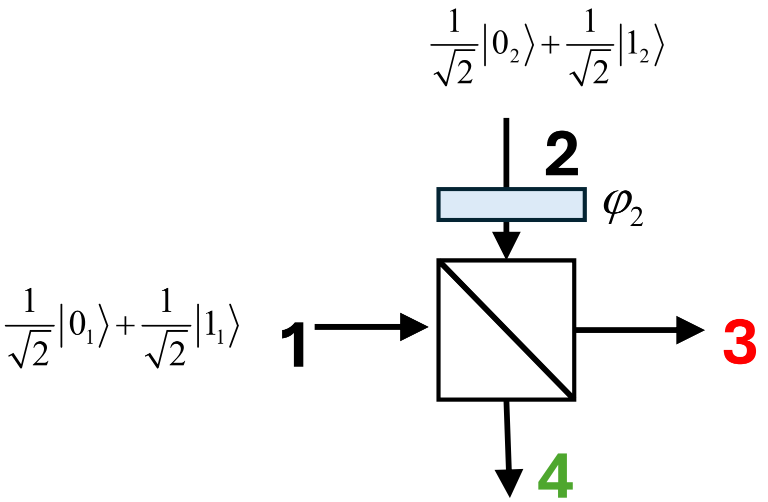


1) Calculez le mode à l'entrée

2) Interprétez les différentes composantes et déterminez lesquelles donnent lieu à une interférence

3) Calculez les modes en sortie et déterminez les nombres de photons en sortie (N3 et N4)

Exercice 10-1: Modèle



$$|IN\rangle=\frac{1}{2}|0_1,0_2\rangle+\frac{1}{2}|1_1,0_2\rangle+\frac{1}{2}|0_1,1_2\rangle+\frac{1}{2}|1_1,1_2\rangle$$

vide — **interférence** — **HOM**

$$|OUT\rangle=\frac{1}{2}\left(1+\left[\tfrac{1}{\sqrt{2}}a_3^++\tfrac{i}{\sqrt{2}}a_4^+\right]+\left[\tfrac{i}{\sqrt{2}}a_3^++\tfrac{1}{\sqrt{2}}a_4^+\right]e^{i\varphi_2}+\left[\tfrac{1}{\sqrt{2}}a_3^++\tfrac{i}{\sqrt{2}}a_4^+\right]\left[\tfrac{i}{\sqrt{2}}a_3^++\tfrac{1}{\sqrt{2}}a_4^+\right]e^{i\varphi_2}\right)\cdot|0_3,0_4\rangle$$

$$|OUT\rangle=\frac{1}{2}\cdot|0_3,0_4\rangle+\frac{e^{i(\varphi_2+\pi/2)/2}}{\sqrt{2}}\cos\left(\frac{\varphi_2}{2}+\frac{\pi}{4}\right)\cdot|1_3,0_4\rangle+\frac{e^{i(\varphi_2+\pi/2)/2}}{\sqrt{2}}\sin\left(\frac{\varphi_2}{2}+\frac{\pi}{4}\right)\cdot|0_3,1_4\rangle+\frac{i\,e^{i\varphi_2}}{2\sqrt{2}}\cdot|2_3,0_4\rangle+\frac{i\,e^{i\varphi_2}}{2\sqrt{2}}\cdot|0_3,2_4\rangle$$

$$P_{00}=\frac{1}{4}$$

$$P_{10}=\frac{1}{2}\cos^2\left(\frac{\varphi_2}{2}+\frac{\pi}{4}\right)\qquad P_{20}=\frac{1}{8}$$

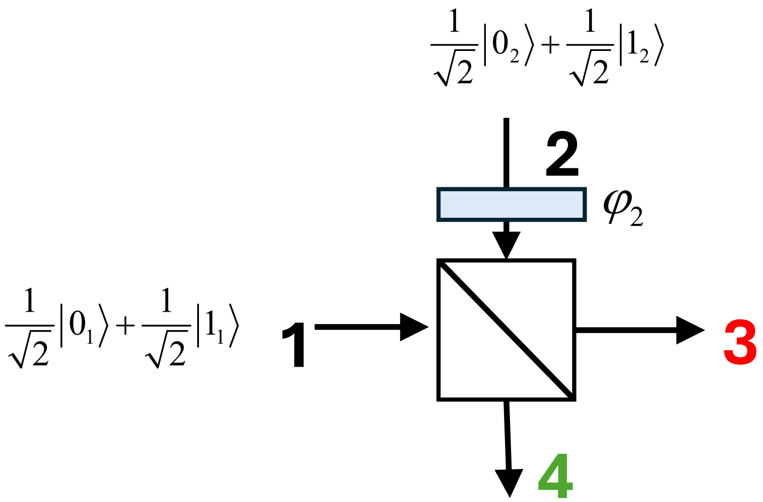
$$P_{01}=\frac{1}{2}\sin^2\left(\frac{\varphi_2}{2}+\frac{\pi}{4}\right)\qquad P_{02}=\frac{1}{8}$$

$\Rightarrow$

$$N3=\frac{1}{4}+\frac{1}{2}\cos^2\left(\frac{\varphi_2}{2}+\frac{\pi}{4}\right)$$

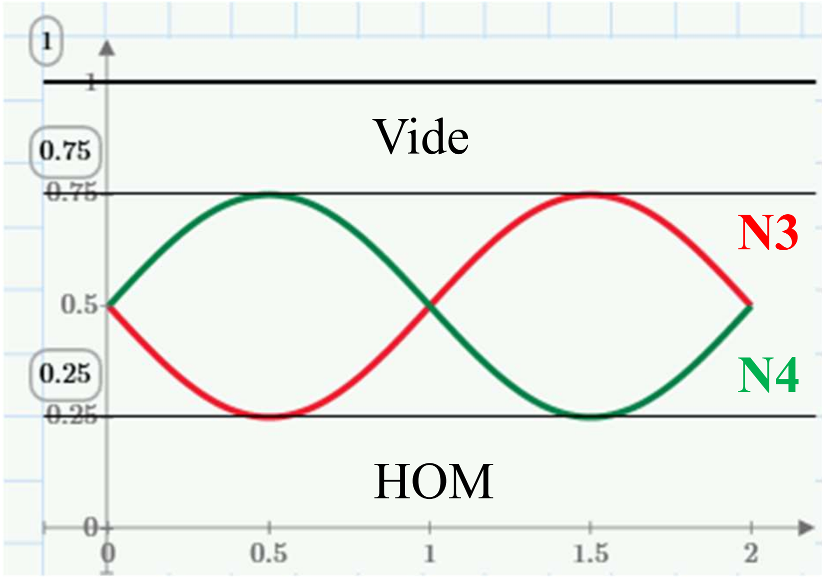
$$N4=\frac{1}{4}+\frac{1}{2}\sin^2\left(\frac{\varphi_2}{2}+\frac{\pi}{4}\right)$$

Exercice 10-1: Modèle

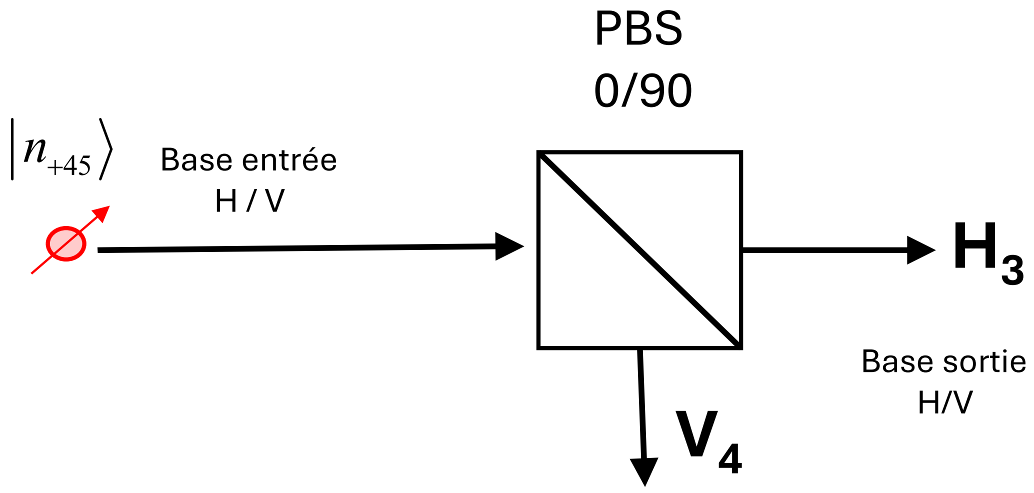


$$|IN\rangle = \frac{1}{2}|0_1,0_2\rangle + \frac{1}{2}|1_1,0_2\rangle + \frac{1}{2}|0_1,1_2\rangle + \frac{1}{2}|1_1,1_2\rangle$$

vide — **interférence** — **HOM**



Exercice 10-2



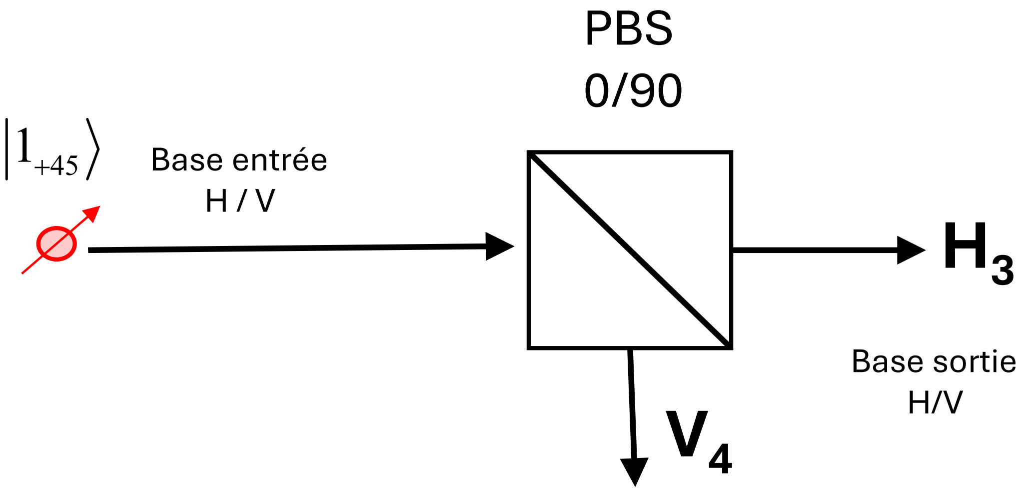
$$|H\rangle \rightarrow 1 \cdot |1_3\rangle$$

$$|V\rangle \rightarrow i \cdot |1_4\rangle$$

1) Calculez le mode superposé en sortie si un seul photon polarisé +45 est injecté en entrée (n=1)

2) Calculez le mode superposé en sortie si deux photons polarisés +45 sont injectés en entrée (n=2)

Exercice 10-2



$$|H\rangle \rightarrow 1\cdot|1_3\rangle$$

$$|V\rangle \rightarrow i\cdot|1_4\rangle$$

$$\Downarrow$$

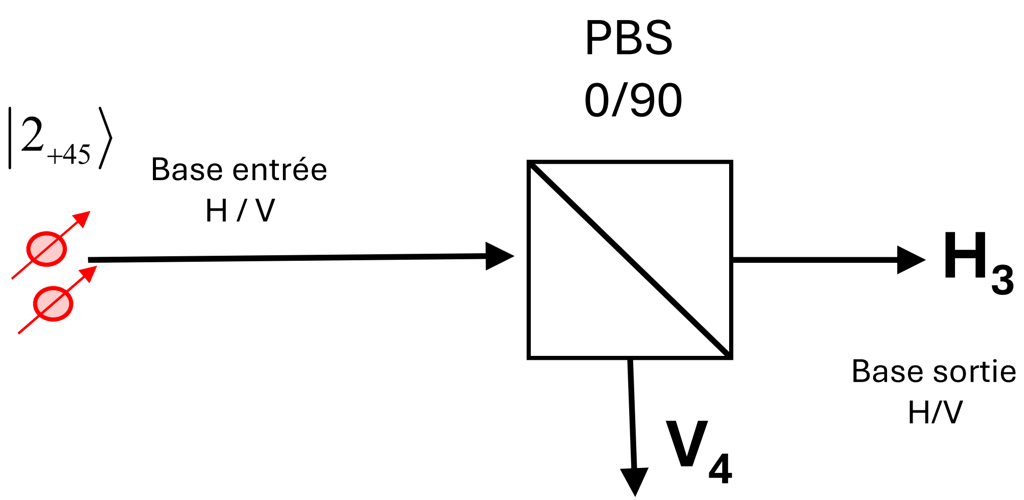
$$a_H^+ \rightarrow 1\cdot a_3^+$$

$$a_V^+ \rightarrow i\cdot a_4^+$$

$$|IN\rangle = a_{+45}^+\cdot|0_H,0_V\rangle = \left(\frac{1}{\sqrt{2}}a_H^+ + \frac{1}{\sqrt{2}}a_V^+\right)\cdot|0_H,0_V\rangle \quad \Rightarrow \quad |OUT\rangle = \left(\frac{1}{\sqrt{2}}a_3^+ + \frac{i}{\sqrt{2}}a_4^+\right)\cdot|0_3,0_4\rangle$$

$$|OUT\rangle = \frac{1}{\sqrt{2}}\cdot|1_3,0_4\rangle + \frac{i}{\sqrt{2}}\cdot|0_3,1_4\rangle$$

Exercice 10-2



$$a_H^+ \rightarrow 1 \cdot a_3^+$$

$$a_V^+ \rightarrow i \cdot a_4^+$$

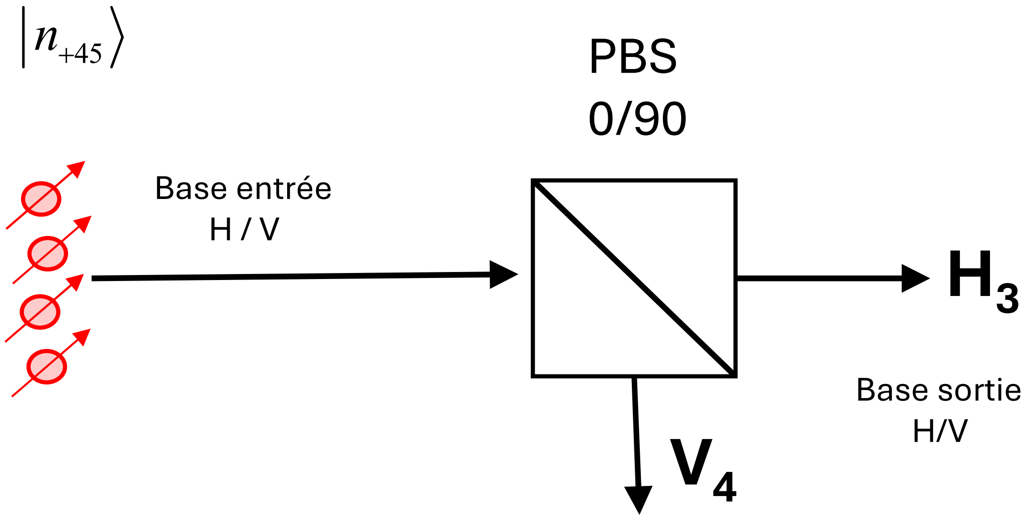
$$|IN\rangle = \frac{\left(a_{+45}^+\right)^2}{\sqrt{2}} \cdot |0_H, 0_V\rangle = \frac{1}{\sqrt{2}} \cdot \left(\left(\tfrac{1}{\sqrt{2}} a_H^+ + \tfrac{1}{\sqrt{2}} a_V^+\right)^2\right) \cdot |0_H, 0_V\rangle = \frac{1}{\sqrt{2}} \cdot \left(\frac{1}{\sqrt{2}} \frac{\left(a_H^+\right)^2}{\sqrt{2}} + a_H^+ a_V^+ + \frac{1}{\sqrt{2}} \frac{\left(a_V^+\right)^2}{\sqrt{2}}\right) \cdot |0_H, 0_V\rangle = \frac{1}{2}|2_H, 0_V\rangle + \frac{1}{\sqrt{2}}|1_H, 1_V\rangle + \frac{1}{2}|0_H, 2_V\rangle$$

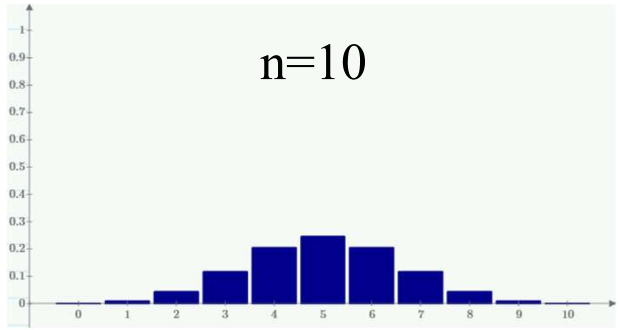
$$\Rightarrow \quad |OUT\rangle = \frac{1}{\sqrt{2}} \cdot \left(\frac{1}{\sqrt{2}} \frac{\left(a_3^+\right)^2}{\sqrt{2}} + a_3^+ \cdot i\, a_4^+ + \frac{1}{\sqrt{2}} \frac{\left(i\, a_4^+\right)^2}{\sqrt{2}}\right) \cdot |0_3, 0_4\rangle$$

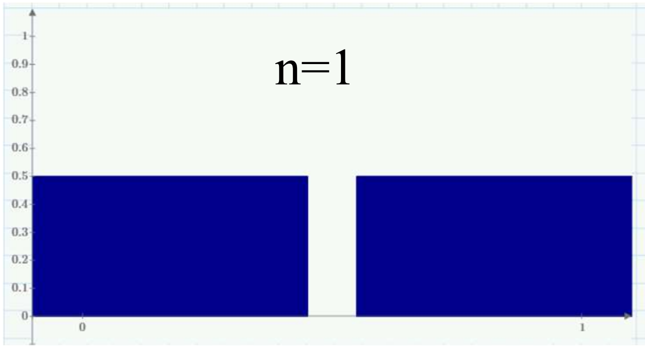
$$\Rightarrow \quad |OUT\rangle = \frac{1}{2} \cdot |2_3, 0_4\rangle + \frac{i}{\sqrt{2}} \cdot |1_3, 1_4\rangle - \frac{1}{2} \cdot |0_3, 2_4\rangle$$

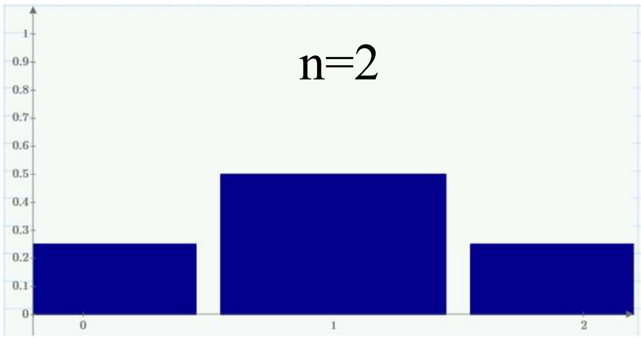
$\Rightarrow$ Distribution binomiale

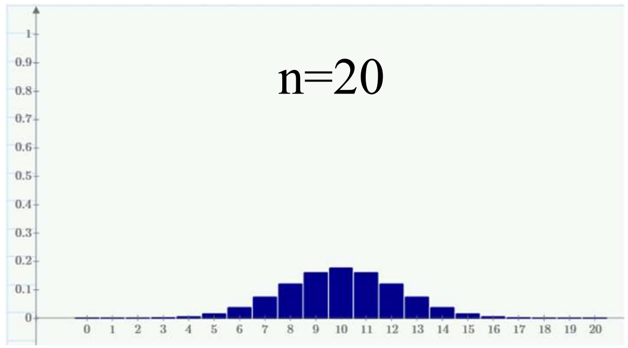
Exercice 10-2



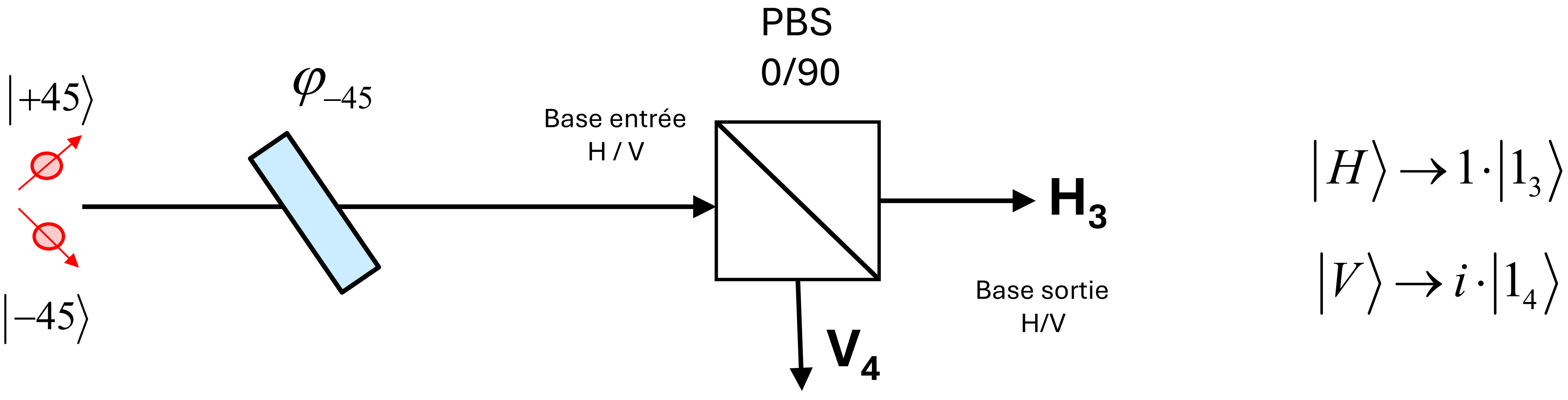








Exercice 10-3

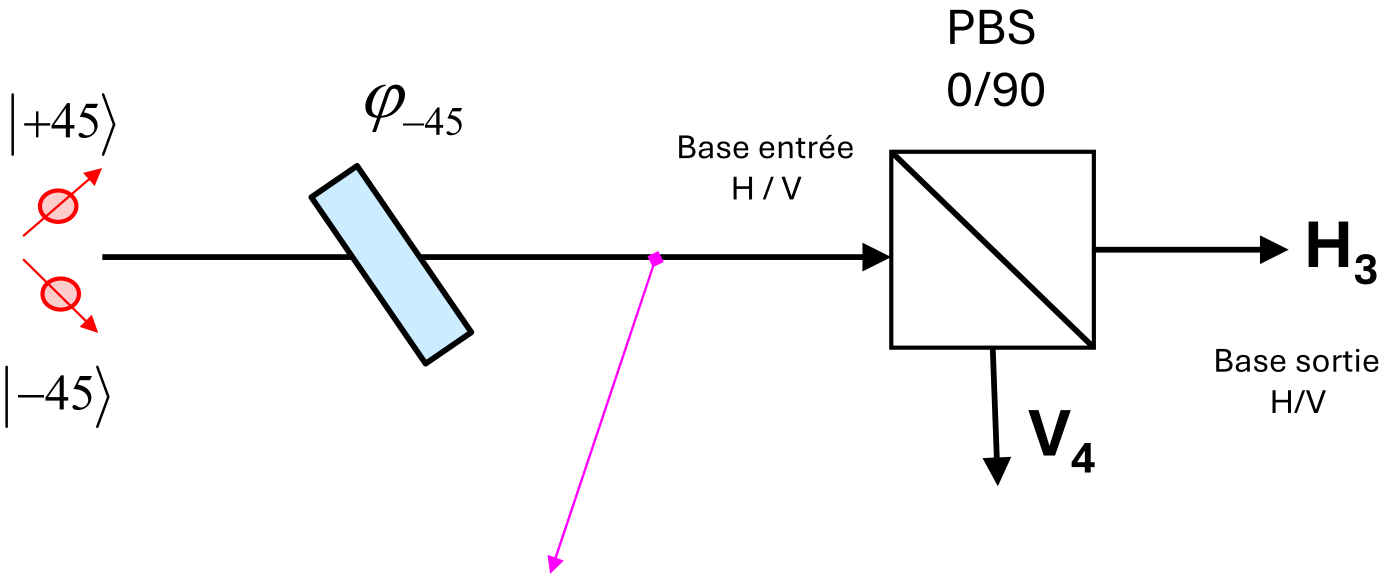


Un lame biréfringente est introduite avec un axe incliné à -45^0.

1) Calculez le mode superposé en sortie si un photon polarisé +45 et un photon polarisé -45 sont injectés simultanément en entrée (n=1) en fonction de la phase φ_{-45} introduite par la lame.

2) qu'attendez-vous pour le cas où un mode cohérent polarisé +45 et un mode cohérent -45 sont injectés avec le même nombre moyen de photons?

Exercice 10-3



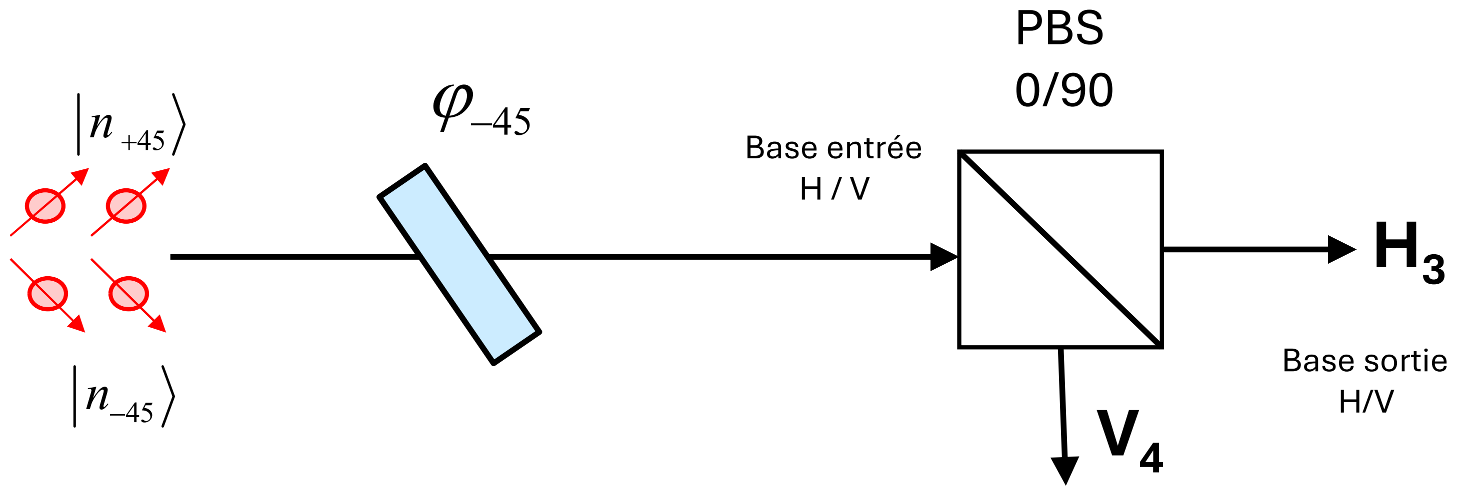
$$a_H^+ \rightarrow 1 \cdot a_3^+$$
$$a_V^+ \rightarrow i \cdot a_4^+$$

$$|IN\rangle = a_{+45}^+ \cdot a_{-45}^+ \cdot e^{i\varphi_{-45}} \cdot |0_H, 0_V\rangle = \left(\tfrac{1}{\sqrt{2}} a_H^+ + \tfrac{1}{\sqrt{2}} a_V^+\right) \cdot \left(\tfrac{1}{\sqrt{2}} a_H^+ - \tfrac{1}{\sqrt{2}} a_V^+\right) \cdot e^{i\varphi_{-45}} \cdot |0_H, 0_V\rangle = \left(\frac{1}{\sqrt{2}} \frac{\left(a_H^+\right)^2}{\sqrt{2}} - \frac{1}{\sqrt{2}} \frac{\left(a_V^+\right)^2}{\sqrt{2}}\right) \cdot e^{i\varphi_{-45}} \cdot |0_H, 0_V\rangle = \frac{e^{i\varphi_{-45}}}{\sqrt{2}} |2_H, 0_V\rangle - \frac{e^{i\varphi_{-45}}}{\sqrt{2}} |0_H, 2_V\rangle$$

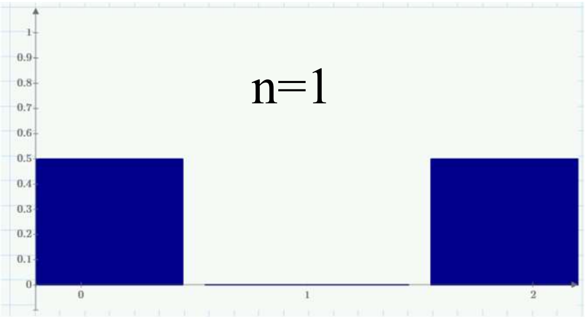
$$\Rightarrow \quad |OUT\rangle = \left(\frac{1}{\sqrt{2}} \frac{\left(a_3^+\right)^2}{\sqrt{2}} - \frac{1}{\sqrt{2}} \frac{\left(i\, a_4^+\right)^2}{\sqrt{2}}\right) \cdot e^{i\varphi_{-45}} \cdot |0_3, 0_4\rangle \quad \Rightarrow \quad |OUT\rangle = \frac{e^{i\varphi_{-45}}}{\sqrt{2}} \left(|2_3, 0_4\rangle + |0_3, 2_4\rangle\right)$$

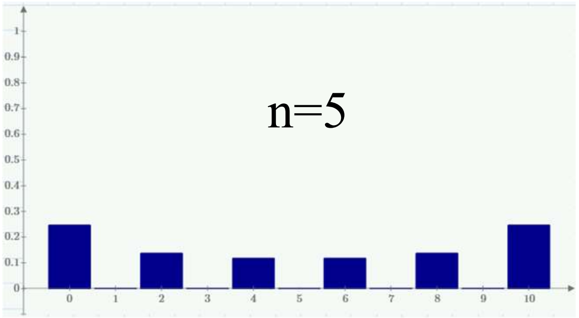
1) Effet HOM: pas de composante $|1_3, 1_4\rangle$
2) Phase globale → aucune rotation de polarisation

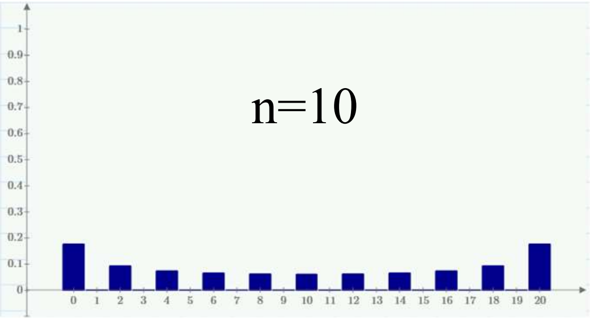
Exercice 10-3

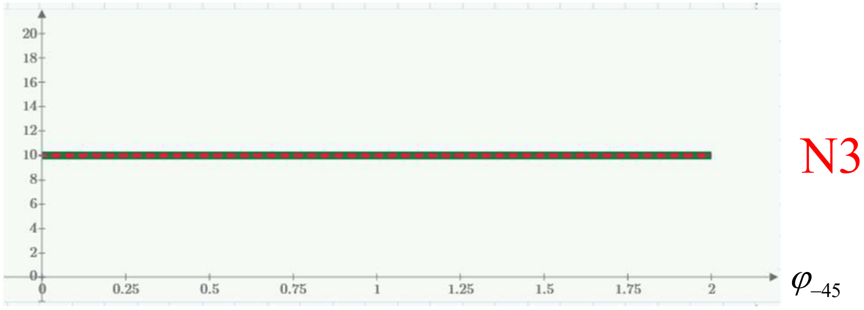


$$a_H^+ \rightarrow 1 \cdot a_3^+$$
$$a_V^+ \rightarrow i \cdot a_4^+$$



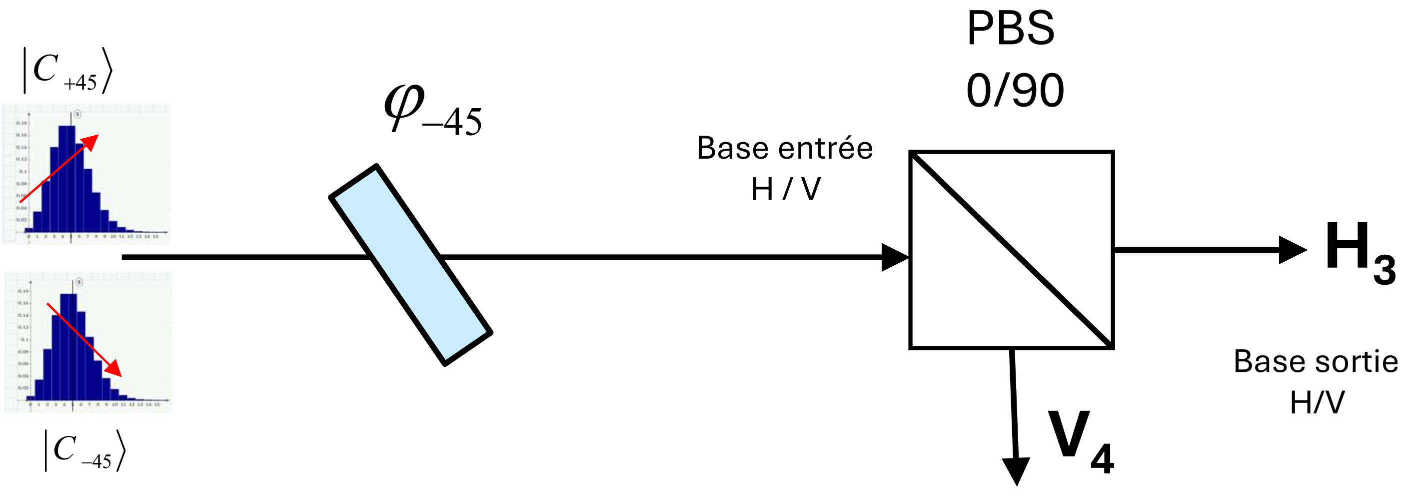






N3=N4=n

Exercice 10-3

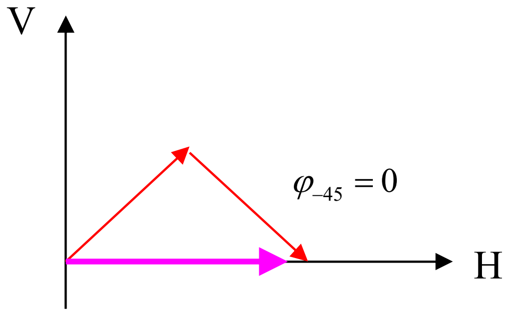


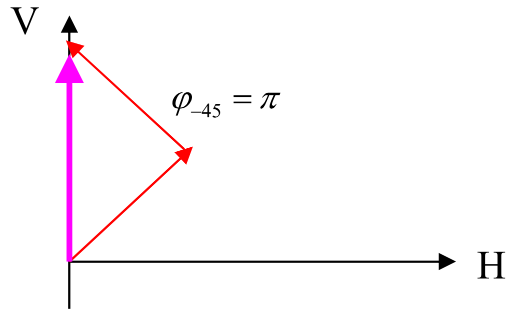
$$a_H^+ \rightarrow 1 \cdot a_3^+$$

$$a_V^+ \rightarrow i \cdot a_4^+$$

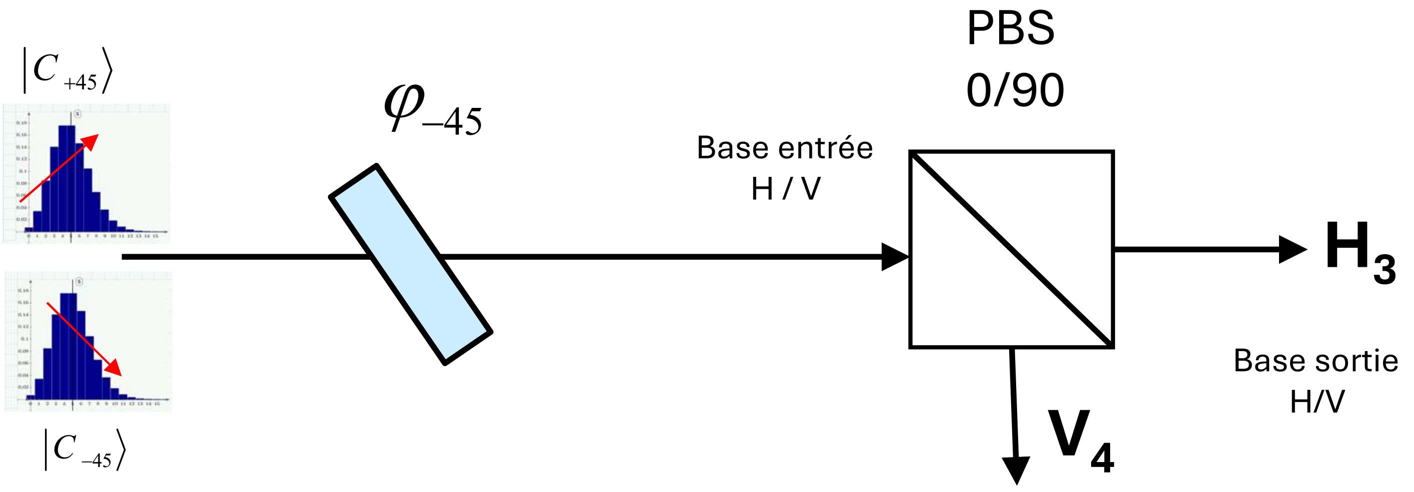
2) qu'attendez-vous pour le cas où un mode cohérent polarisé +45 et un mode cohérent -45 sont injectés avec le même nombre moyen de photons?

Les modes cohérents se comportent comme des lasers classiques → la polarisation doit tourner





Exercice 10-3



$$a_H^+ \rightarrow 1 \cdot a_3^+$$

$$a_V^+ \rightarrow i \cdot a_4^+$$